PHYS 231 – Dec. 6, 2023

Today: Binary Numbers & Review.

Reminder: Submit final project lab notebooks by the <u>end</u> of tomorrow's lab.

Ordinary base-10 number system,
have 10 digits 0, 1, 2, ..., 9.

Consider an arbitrary base-10 number

2376901

$$= 2\times10^6 + 3\times10^5 + 7\times10^4 + 6\times10^3 + 9\times10^2 + 0\times10^1 + 1\times10^0$$

By analogy, the base-2 or binary number system has 2 digits: 0 1

An arbitrary base-2 number might be

1011001

can express this number as

$1011001_2 =$ ← base 2

$1\times2^6 + 0\times2^5 + 1\times2^4 + 1\times2^3 + 0\times2^2 + 0\times2^1 + 1\times2^0$

$= 64 + 16 + 8 + 1 = 89_{10}$

↑ base 10

Convert 189_{10} to base-2 system.

$2^0 = 1$

× $2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

× $2^6 = 64$

$2^7 = 128$

× $2^8 = 256$

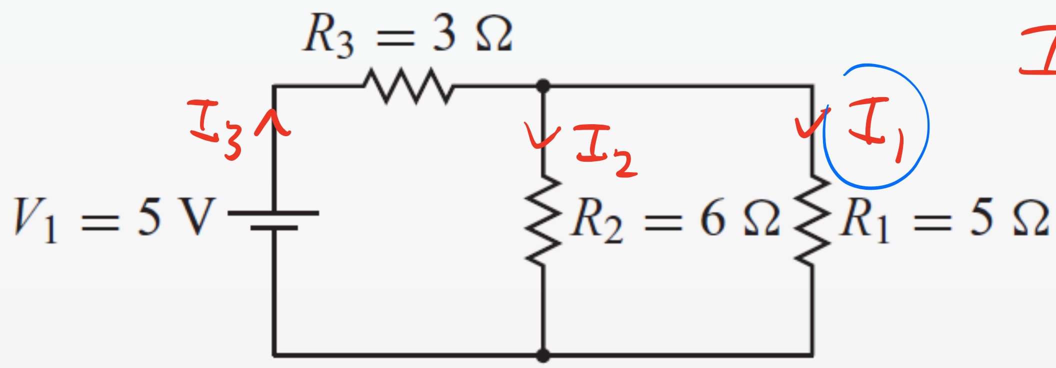
189

192

$$\begin{array}{ccccccccc} 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 2^8 & 2^7 & \dots & & & & & & 2^0 \end{array}$$

$$\boxed{10111101_2 = 189_{10}}$$

Eggleston Chapter 1 Problem 5

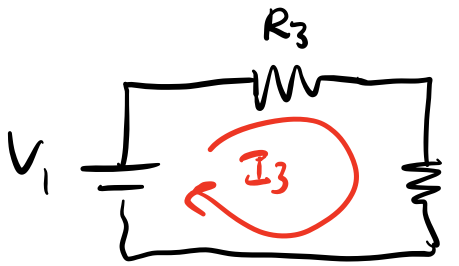


$I_3 = I_1 + I_2$

Figure 1.31 Circuit for Problems 4 and 5.

5. Compute the current through R_1 ~~and R_2~~ of Fig. 1.31.

Combine R_2 & R_1 in parallel



$$R_{\parallel} = \frac{R_1 R_2}{R_1 + R_2}$$

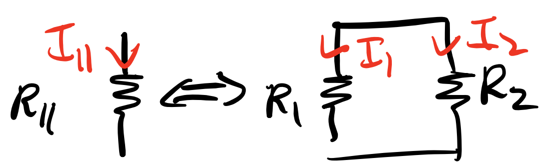
Components in series have same current.

$$I_3 = \frac{V_1}{R_3 + R_{\parallel}}$$

Voltage across $R_{\parallel}$ is $I_3 R_{\parallel}$

$$V_{\parallel} = \frac{V_1}{R_3 + R_{\parallel}} R_{\parallel} \quad *$$

For components in parallel, all voltages are the same.



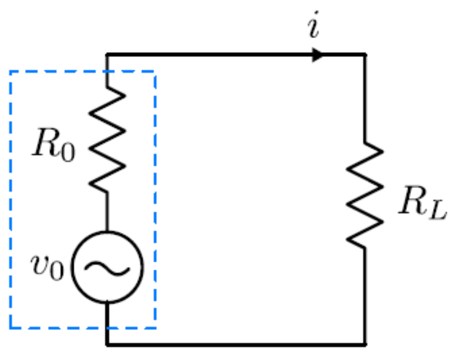
$\therefore$ $V_{11} = V_{R_1}$ (voltage across R_1).

$$\therefore V_{11} = I_1 R_1 \Rightarrow I_1 = \frac{V_{11}}{R_1}$$

$$\boxed{I_1 = \frac{V_1}{R_1} \frac{R_{11}}{R_3 + R_{11}}}$$

Assignment #2 Problem 2

2. Function generators, like the one you use in the lab, have an internal resistance R_0 in series with the output of the generator. For example, the high-resistance output of the PASCO PI–9587C is 600 Ω. The figure below shows a schematic of a function generator (inside the blue dashed line) connected across a load resistance R_L.



(a) For what value of R_L is the power dissipated by the load resistance a maximum? Express your answer in terms of R_0. That is, $R_L = c\,R_0$. Find the appropriate numerical value of the constant c. You must show your work/reasoning to earn full credit.

$$P_L = i^2 R_L \qquad i = \frac{V_0}{R_0+R_L}$$

$$\therefore P_L = \frac{V_0^2 \, R_L}{(R_0+R_L)^2}$$

for both $R_L \to 0$ & $R_L \to \infty$, P_L is zero. So, there must a max. in between $0 < R_L < \infty$

To maximize P_L w.r.t. R_L, find $\frac{dP_L}{dR_L} = 0$

$$\frac{dP_L}{dR_L} = \frac{\overset{1}{\cancel{V_0^2}}}{\cancel{(R_0+R_L)}^2} + \frac{\cancel{V_0^2} R_L\,(-2)}{(R_0+R_L)^3} = 0$$

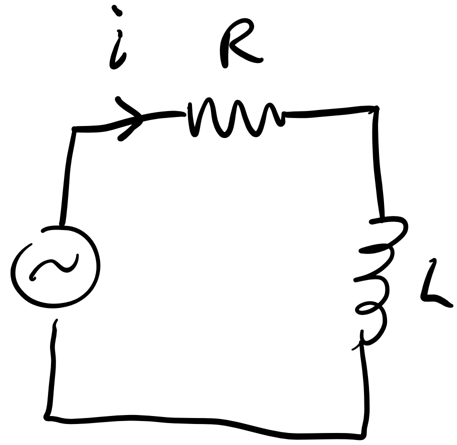
$$1 = \frac{2R_L}{R_0 + R_L} \Rightarrow \boxed{R_L = R_0}$$

impedance matching gives max power to R_L.

Assignment #3 Extra Practice

- Consider a resistor R in series with an inductor L driven by a function generator that outputs $v_{\text{in}} = V_0 \sin \omega t$. The current in the circuit is given by $i = I_0 \sin(\omega t + \phi)$. Find I_0 and ϕ.

$v_{in} = V_0 e^{j\omega t}$

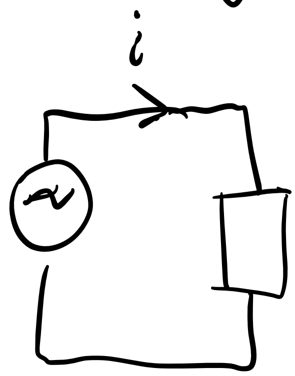


$i = I_0 e^{j(\omega t + \phi)}$

find I_0 & ϕ.

$\Downarrow$

v_{in}



$Z = Z_R + Z_L = R + j\omega L$

Important results: $I_0 = \dfrac{V_0}{|Z|}$

$$\tan\phi = -\frac{Im[Z]}{Re[Z]}$$

For our circuit $Z = R + j\omega L$ $\quad Re[Z] = R$

$Im[Z] = \omega L$

$$\boxed{\tan\phi = -\frac{\omega L}{R}}$$

$$|Z|^2 = ZZ^*$$

$$= (R + j\omega L)(R - j\omega L)$$

$$= R^2 + (\omega L)^2$$

$$|Z| = \sqrt{R^2 + (\omega L)^2}$$

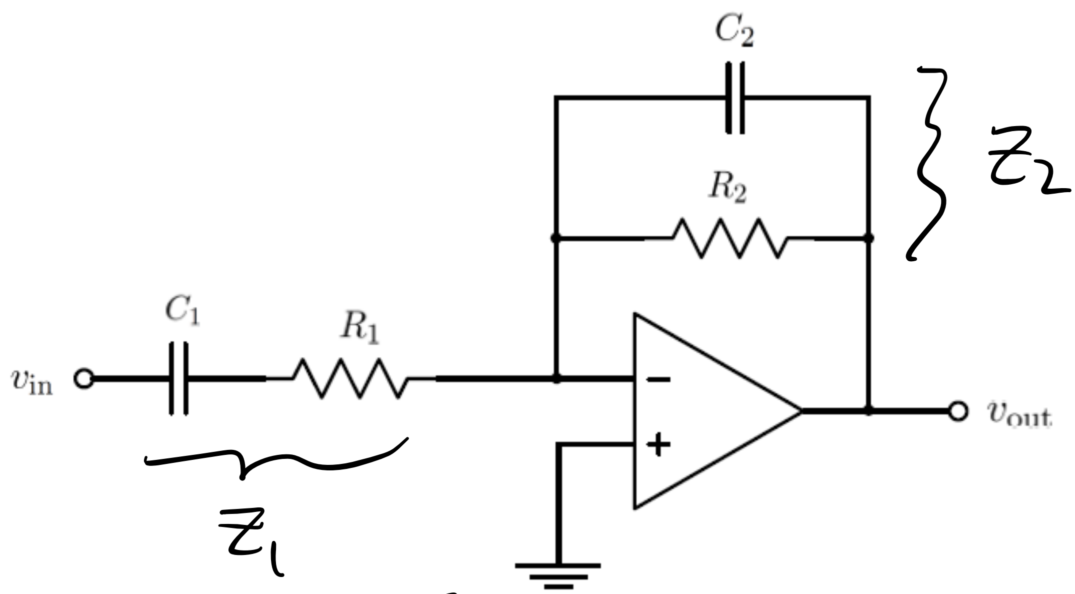
$$I_0 = \frac{V_0}{\sqrt{R^2 + (\omega L)^2}} = \frac{V_0 / R}{\sqrt{1 + (\omega \frac{L}{R})^2}}$$ (low-pass filter)

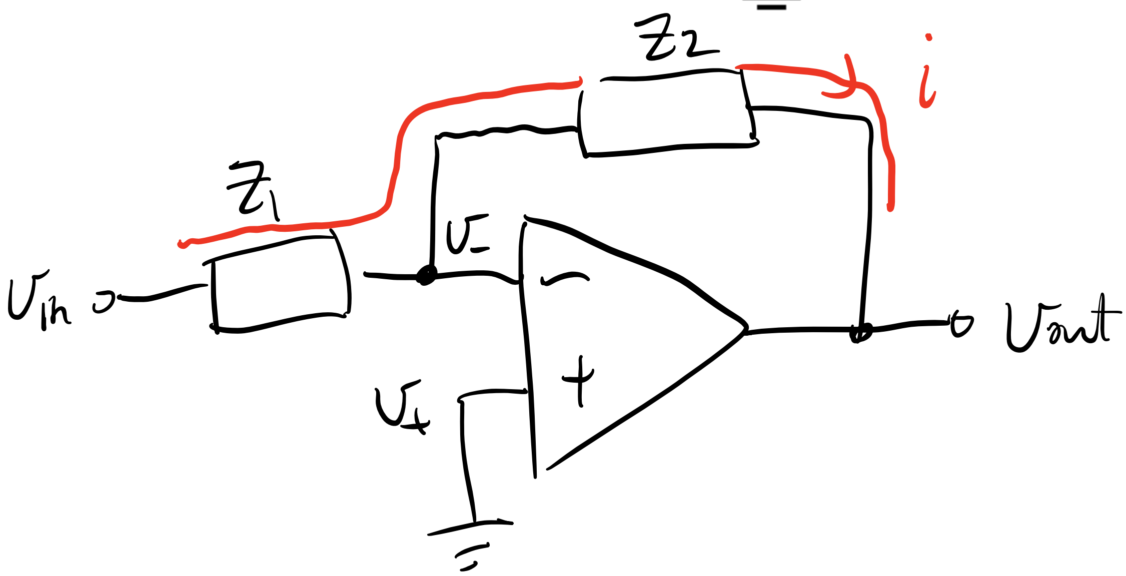
Assignment #4 Extra Practice

1. The circuit below is a *bandpass* filter/amplifier. Show that:

$$\left|\frac{v_{\text{out}}}{v_{\text{in}}}\right| = G\frac{\omega R_1 C_1}{\sqrt{1+(\omega R_1 C_1)^2}}\frac{1}{\sqrt{1+(\omega R_2 C_2)^2}}.$$

Assume that v_{in} is a sinusoidal input. When analysing the circuit, make use of the fact that $Z_C = 1/(j\omega C)$ and $v_C = iZ_C$. What is G in terms of the circuit components?





inverting amplifier

$$V_{out} = -\frac{Z_2}{Z_1}V_{in}$$

GR #1 : no current into/out of op amp inputs.

GR#2 : w/ neg. feedback, require $V_+ \approx V_-$

Here $v_+ = 0 \quad \therefore v_- = 0.$

$$v_{in} - iZ_1 = v_- = 0 \Rightarrow i = \frac{v_{in}}{Z_1}$$

$$\cancel{v_-}^{\,0} - iZ_2 = v_{out}$$

$$\therefore v_{out} = -iZ_2$$

$$\boxed{v_{out} = -v_{in}\frac{Z_2}{Z_1}}$$

$$\therefore \left|\frac{v_{out}}{v_{in}}\right| = \left|\frac{Z_2}{Z_1}\right| = \frac{|Z_2|}{|Z_1|}$$

$$Z_1 = R_1 + \frac{1}{j\omega C_1} = \frac{1 + j\omega R_1 C_1}{j\omega C_1}\left(\frac{R_1}{R_1}\right)$$

$$Z_1 = \frac{1 + j\omega R_1 C_1}{j\omega R_1 C_1} R_1$$

$$|Z_1|^2 = Z_1 Z^* = \left(R_1 \frac{1 + j\omega R_1 C_1}{j\omega R_1 C_1}\right)\left(R_1 \frac{1 - j\omega R_1 C_1}{-j\omega R_1 C_1}\right)$$

$$|Z_1|^2 = R_1^2 \frac{1+(\omega R_1 C_1)^2}{(\omega R_1 C_1)^2}$$

$$|Z_1| = R_1 \frac{\sqrt{1+(\omega R_1 C_1)^2}}{\omega R_1 C_1}$$

$$Z_2 = R_2 \parallel Z_{C_2} = \frac{R_2 Z_{C_2}}{R_2 + Z_{C_2}} = \frac{R_2 \frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}}$$

$$= \frac{R_2}{1 + j\omega R_2 C_2}$$

$$|Z_2| = \sqrt{Z_2 Z_2^*} = \frac{R_2}{\sqrt{1+(\omega R_2 C_2)^2}}$$

Finally: $\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{Z_2}{Z_1}\right| = \frac{|Z_2|}{|Z_1|}$

$$= \frac{R_2}{\sqrt{1+(\omega R_2 C_2)^2}} \frac{\omega R_1 C_1}{R_1 \sqrt{1+(\omega R_1 C_1)^2}}$$

$$\therefore \left|\frac{V_{out}}{V_{in}}\right| = \underbrace{\frac{R_2}{R_1}}_{\substack{G \\ \text{gain factor}}} \underbrace{\frac{\omega R_1 C_1}{\sqrt{1+(\omega R_1 C_1)^2}}}_{\substack{\text{high-pass} \\ \text{filter}}} \underbrace{\frac{1}{\sqrt{1+(\omega R_2 C_2)^2}}}_{\substack{\text{low-pass} \\ \text{filter}}}$$

Eggleston Chapter 8 Problem 2

2. Using only NAND gates, construct a circuit that will implement the following logical expressions. Use Boolean algebra to simplify the expressions as much as possible before you begin.

(a) $(A \cdot B) + (\overline{A} \cdot B) + (A \cdot \overline{B}) + (\overline{A} \cdot \overline{B})$

(b) $[(A \cdot B) + C] \cdot [(A \cdot B) + D]$

(c) $[(\overline{A} \cdot B) \cdot (A \cdot B)] + (A \cdot B)$

(d) $(1 + B) \cdot (A \cdot B \cdot C)$

(a)

$$Q = (A \cdot B) + (\bar{A} \cdot B) + (A \cdot \bar{B}) + (\bar{A} \cdot \bar{B})$$

From de Morgan, know

$$\overline{A \cdot B} = \bar{A} + \bar{B}$$

$$\overline{A + B} = \bar{A} \cdot \bar{B}$$

Note: $A \cdot B + A \cdot \bar{B}$

$$= A \cdot \underbrace{(B + \bar{B})}_{1} = A \cdot 1 = A$$

likewise

$$\bar{A} \cdot B + \bar{A} \cdot \bar{B} = \bar{A} \cdot \underbrace{(B + \bar{B})}_{1} = \bar{A}$$

$$\therefore Q = A + \bar{A} = 1.$$

$$\boxed{\therefore Q = 1 \text{ always}}$$

check:

A	B	$\overline{A}$	$\overline{B}$	$A\cdot B$	$A\cdot\overline{B}$	$\overline{A}\cdot B$	$\overline{A}\cdot\overline{B}$	Q
0	0	1	1	0	0	0	1	1
0	1	1	0	0	0	1	0	1
1	0	0	1	0	1	0	0	1
1	1	0	0	1	0	0	0	1

(b) $Q = [(A\cdot B)+C]\cdot[(A\cdot B)+D]$

$$= \underbrace{A\cdot B\cdot A\cdot B}_{\substack{A\cdot A\cdot B\cdot B \\ = A\cdot B}} + A\cdot B\cdot D + C\cdot A\cdot B + C\cdot D$$

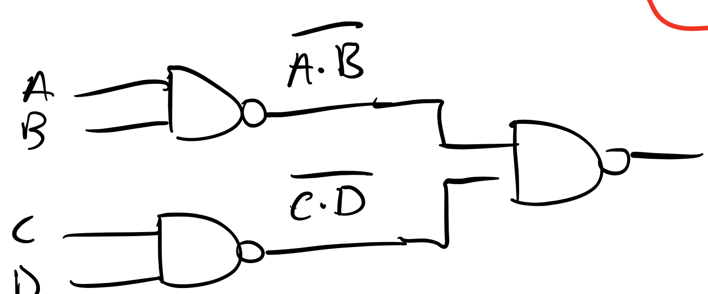
$$= A\cdot B\cdot(1 + D + C) + C\cdot D$$

$1 + \text{anything} = 1$

$$\therefore Q = A\cdot B + C\cdot D = \overline{\overline{A\cdot B}} + \overline{\overline{C\cdot D}}$$

by deMorgan

$$= \overline{\overline{A\cdot B}\cdot\overline{C\cdot D}}$$



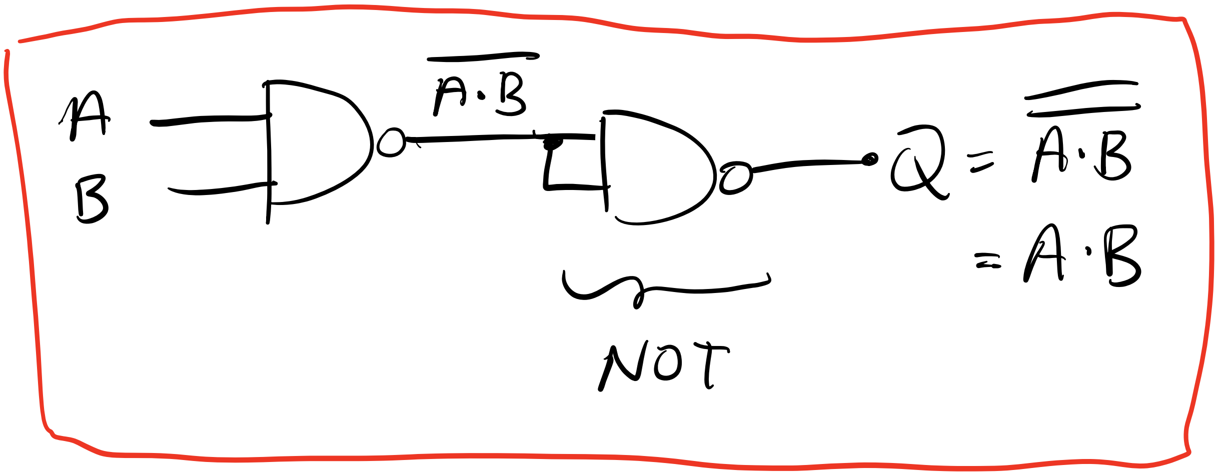
$Q = \overline{\overline{A\cdot B}\cdot\overline{C\cdot D}}$

(c) $$Q = \underbrace{\left[(\bar{A}\cdot B)\cdot(A\cdot B) \right]} + (A\cdot B)$$

$$= \underbrace{\bar{A}\cdot A}_{0} \cdot \underbrace{B\cdot B}_{B}$$

$$= 0\cdot B = 0$$

$$\therefore Q = 0 + A\cdot B = A\cdot B = \overline{\overline{A\cdot B}}$$



(d) $$Q = (1 + B)\cdot(A\cdot B\cdot C)$$

1 + anything = 1

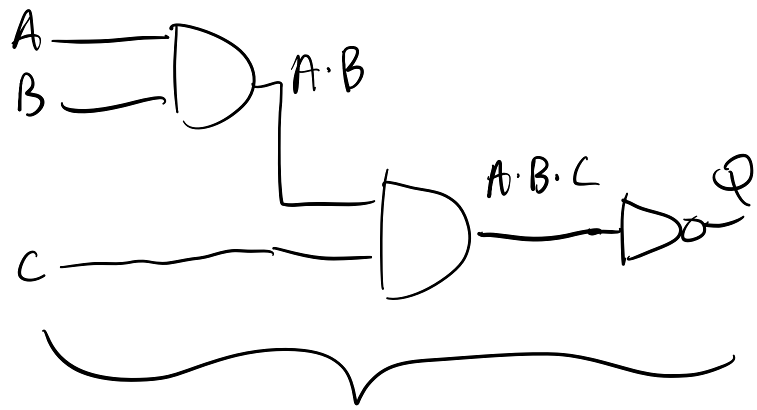
$1 \cdot X = 1$ where X can be anything (LO or HI)

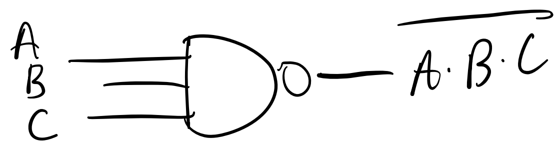
$$\therefore Q = 1\cdot(A\cdot B\cdot C) = A\cdot B\cdot C$$

Note: 3-input NAND truth table

A	B	C	Q
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

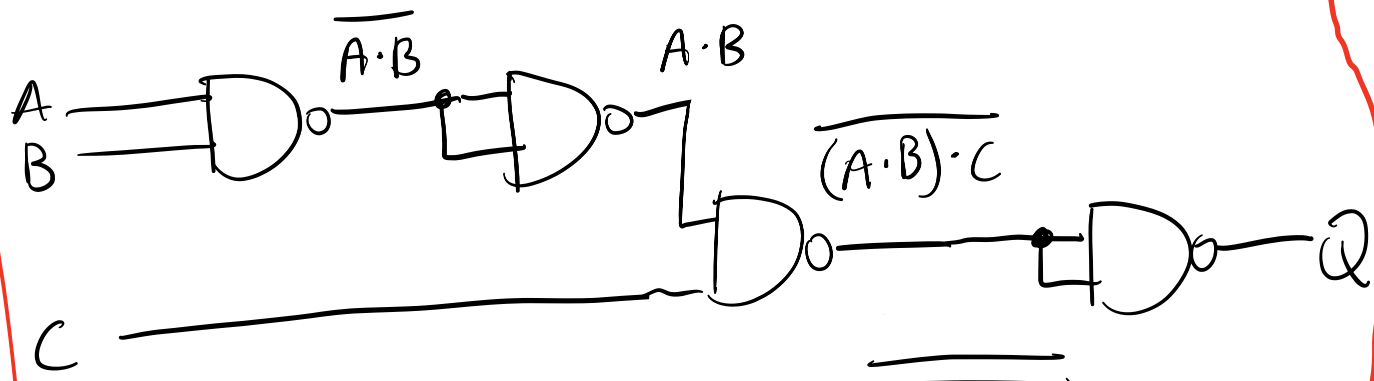
consider





A	B	C	A·B	A·B·C	Q
0	0	0	0	0	1
0	0	1	0	0	1
0	1	0	0	0	1
0	1	1	0	0	1
1	0	0	0	0	1
1	0	1	0	0	1
1	1	0	1	0	1
1	1	1	1	1	0

same



$$Q = \overline{\overline{(A\cdot B)\cdot C}}$$

$$= (A\cdot B)\cdot C$$

$$= A\cdot B\cdot C \quad ✓$$